April 2008

Central European University

Advanced Time Series Analysis

2 Hours

Answer ANY THREE questions, out of 7.

1.

a) Carefully define the concepts of strict stationarity, ergodicity and mixing.

b) Show that an identically and independently distributed (i.i.d.) process has all of the properties listed in part (a).

c) What is a white noise process? Is a white noise process always i.i.d.? Is an i.i.d. process always white noise? Explain your answers.

d) Consider a linear process, having the form $x_t = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j}$ where $\{\theta_0, \theta_1, \theta_2, \ldots\}$ is a constant sequence and $\{\varepsilon_t\}$ is i.i.d. with mean zero and variance σ^2. Is such a process always (i) strictly stationary? (ii) wide-sense stationary? If the answer to the last question is "No", can you state a further condition under which wide-sense stationarity holds?

2. Answer the following questions concerning the second-order autoregressive model

$$x_t = 0.32 + 0.4x_{t-1} - 0.04x_{t-2} + \varepsilon_t \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

a) Is it stable? If the sequence $\{\varepsilon_t\}$ is replaced by zeros for an infinite number of periods, to what value (if any) does x_t converge?

b) What are the characteristics of the infinite-order moving average representation $x_t = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j}$? Is the sequence $\{\theta_0, \theta_1, \theta_2, \ldots\}$ monotonic or sinusoidal? Explain.

c) Obtain the first and second order autocorrelations. What can be said about the sequence of higher order autocorrelations?

d) Briefly discuss the estimation of this model.

3. Consider the ARMA(1,1) model

$$x_t = \alpha + \lambda_1 x_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \qquad |\lambda_1| < 1$$

where $\{\varepsilon_t\}$ is assumed to be i.i.d. with mean zero and variance σ^2.

a) What do you mean by quasi-maximum likelihood (QML) estimation of α, λ_1, θ_1 and σ^2 in this model? Explain how the estimators could be computed.

b) What are the known properties of the QML estimator? How do these differ from the properties of true maximum likelihood?

c) What problems arise in the case $\theta_1 = -\lambda_1$?

4.

a) How could you test for the presence of conditional heteroscedasticity in a time series?

b) Consider the GARCH(1,1) model

$$x_t = \varepsilon_t \sigma_t$$
$$\sigma_t^2 = \omega + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$
$$\varepsilon_t \sim iid(0,1), \quad \alpha_1 > 0, \; \beta_1 > 0$$

What are the properties of the process x_t? Is it white noise? Is it wide-sense stationary? State any additional assumptions you may need to make.

c) Show that the GARCH(1,1) model also has a representation as an ARMA(1,1), being careful to comment on the properties of the "error term".

d) How (briefly) would you estimate the model in part (b)?

5.

a) What fundamental theorems from asymptotic distribution theory are used in the derivation of tests for a unit root?

b) Consider testing the hypothesis $\lambda = 1$ in the model

$$x_t = \lambda x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

Describe a test of this hypothesis, and show how the results you mentioned in part (a) help to determine the asymptotic distribution under the null.

c) Show that the test statistic diverges as the sample size goes to infinity when $\lambda < 1$, and hence that the test is consistent.

d) Briefly explain how the test can be adapted to deal with the case where ε_t is not i.i.d., but is a stationary invertible ARMA process.

6.

a) Explain carefully what it means to you to say that "a time series is I(0)". Is there a single agreed definition of this term? What are the issues arising in constructing a useful definition?

b) Describe a test of I(0). Give a brief explanation of the assumed asymptotic distribution under the null hypothesis, and explain the link between this distribution and the conditions that define "I(0)", as discussed in part (a).

7. You are asked to construct forecasts of an arbitrary univariate time series.

a) Briefly explain how you might formulate and estimate a forecasting model.

b) What statistical characteristics should the series possess, to ensure validity of the techniques you propose?

c) Explain how you would choose between alternative models.

d) What statistical characteristics should the series possess (possibly additional to those given in (b)) to allow valid 95% confidence intervals for the forecasts to be constructed?