Central European University

Advanced Time Series Analysis

Exercise 1: Stochastic Sequences

Note: Possible correct answers to the questions in 1, 2 and 3 are "Yes", "No", and "Uncertain" (not enough information to say). Give the reasons for your answers.

1. Suppose $\{x_t,\ t = 1,2,3,\ldots\}$ is a wide-sense stationary random sequence. Are the following sequences wide-sense stationary?
   (a) $\{x_t + \cos at,\ t = 1,2,3,\ldots\}$ a constant.
   (b) $\{x_t + x_1,\ t = 1,2,3,\ldots\}$.
   (c) $\{x_t^2,\ t = 1,2,3,\ldots\}$

2. Consider the random sequence $\{x_t,\ t = 1,2,3,\ldots\}$ where $x_t = (-1)^t X$, and $X \sim N(0,1)$. Is this sequence
   (a) stationary?
   (b) ergodic?
   (c) mixing?

3. Consider the random sequence $\{\varepsilon_t\varepsilon_{t-1},\ t = 1,2,\ldots\}$ where $\varepsilon_t \sim NI(0,\sigma^2)$ for $t \geq 0$ (in other words, ε_t is i.i.d. and normally distributed, with mean zero). Is this sequence
   (a) stationary?
   (b) serially uncorrelated (white noise)?
   (c) independent and identically distributed (i.i.d.)? (Hint: remember,$E(\varepsilon_t^4) = 3\sigma^4$).
   (d) mixing?
   (e) ergodic?

4. If $x_t = \sum_{j=0}^{\infty} \lambda^j \varepsilon_{t-j}$ where $|\lambda| < 1$ and $\varepsilon_t \sim NI(0,\sigma^2)$, find formulae for the autocovariances,

$$\gamma_m = E(x_t x_{t+m})$$

   for $j = 0,1,2,3,\ldots$ (there is a pattern). Is this a mixing process?