Central European University

Advanced Time Series Analysis

Exercise 2: Difference Equations

1. Solve the following difference equations for $t = 1, 2, 3, \ldots$ Discuss properties of each case as $t \to \infty$
   a) $x_t = \alpha + \beta x_{t-1}$ where $|\beta| < 1$ and $x_0 = A$.
   b) $x_t = \alpha + \beta x_{t-1} + \gamma t$ where $|\beta| < 1$ and $x_0 = 0$.

2. Find the roots, and hence check the stability, of the following second order difference equations.
   a) $x_t = x_{t-1} - 0.25x_{t-2}$
   b) $x_t = x_{t-1} - 0.24x_{t-2}$
   c) $x_t = x_{t-1} - 0.26x_{t-2}$

3. Consider the second order stochastic difference equation
   $$x_t = \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \varepsilon_t \quad \varepsilon_t \sim \text{iid}(0, \sigma^2)$$
   In the solution (moving average form) of the equation,
   $$x_t = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j}$$
   show that
   a) In the case $\lambda_1 = 1,\ \lambda_2 = -0.25$, the coefficients θ_j decline monotonically to zero.
   b) In the case $\lambda_1 = 1,\ \lambda_2 = -0.26$, the θ_j follow a sinusoidal pattern.
   c) In case (b), after how many lags do the coefficients change sign?

4. Using the Yule-Walker equations, determine the autocorrelations ρ_1, ρ_2 and ρ_3 for
   a) Case a) of Question 3.
   b) Case b) of Question 3.