Central European University

Advanced Time Series Analysis

Exercise 5 - A Simulation Experiment

In this exercise you use the TSM package to study the distribution of the coefficient in a first-order autoregressive model,

$$x_t = \alpha + \rho x_{t-1} + u_t$$

where $u_t \sim N(0,1)$, for different sample sizes and parameter values. For example, try $T = 50$ and $T = 500$. For each of these cases, first set $\alpha = 0$ and (for example) $\rho = 0.3$, $\rho = 0.8$ and $\rho = 1$. Then repeat the experiments with $\alpha = 5$, giving a total of 12 experiments. Your findings may suggest other cases to look at.

Procedure

To run the exercise, download the file TSAEx5.tsm from the web page to your work space. Either start TSM from the Start Menu and load the settings with the command File / Settings / Open, or simply double-click the .tsm file in Windows Explorer to start TSM directly.

The file contains the main settings required, including a baseline model with specifications $T = 50$ and $\rho = \alpha = 0$, stored as AR1_T50_r00_a00. To load this model into the program, choose the menu item Setup / Model Manager (or simply press the toolbar button) and double-click the model name in the list to load it. Note that for estimation purposes the first 100 generated values are discarded to eliminate possible start-up effects, so the sample runs from 101 to 150.

To see how the model is specified, click on  to open the linear regression dialog. Note that Xvariable is selected both as dependent variable and as a regressor of "Type 2". A single lag is specified on the scrollbar. The program automatically removes the current value of a Type 2 regressor matching the dependent variable, so this setup specifies an autoregression. To specify an AR(p) model, simply move the lag scrollbar to the desired value of p.

To simulate the model once and view the time plot, press the assignable  toolbar button, which has been assigned to the command Actions / Simulate Current Model. The simulated series in this case replaces the existing series in the data set. To estimate the model on this new series either press the  button, or press Go in the linear regression dialog.

To run a Monte Carlo experiment, either select Setup / MonteCarlo Experiment or press the assignable  toolbar button, which has been assigned to this dialog. When the dialog opens, press Run to perform 5000 replications. When the run concludes, a dialog box opens with commands to view the results in graphical form. This dialog can also be opened from Graphics / Monte Carlo Distributions. The experimental results are saved with the model, in a file called AR1_T50_r00_a00.tsd. You have only to re-load the model to view the results again at a later date

To simulate different cases, store each specification as a new model. The experimental

results will then be saved for future study. For example, to create the model with $T = 500$, $\rho = 0.8$ and $\alpha = 5$, do as follows.

1. Open the Model Manager and load the existing model AR1_T50_r0_a0.
2. To change the parameters, select Values / Equation. Enter the value 5 in the field labelled Intercept and 0.8 in the field labelled [2]Xvariable(-1).
3. To change the sample size, Select Setup / Set Sample and move the Last Observation scrollbar to 500.
4. In the Model Manager click Store Current Model, edit the name field to read AR1_T500_r08_a50, and click OK.
5. Open the Monte Carlo dialog. Choose the radio button Data Generation Model, deselect the currently highlighted model and select AR1_T500_r08_a50. Then choose Estimation Model, and repeat the procedure. (Clicking an item either selects it or deselects it, depending on its current status.)
6. Click Run as before.

To increase the number of replications for more accurate results, move the scrollbar. Simply pressing Run a second time lets you extend the current experiment, provided no settings are changed.

To experiment with even larger sample sizes you will need to extend the data space, as follows.

1. Select Setup / Data Transformation and Editing, from the menu bar (or press f(x) on the tool bar).
2. Find the option Resize Sample in the choice widget.
3. Enter the desired sample size in the text field. and press Go.

Note that a placeholder variable Zeros is created. This can be deleted once the extended series has been created.

Discussion

The main points of interest here are the distributions of the slope and intercept coefficients, and also those of the corresponding '*t*-ratios'. When $|\rho| < 1$, these are both normally distributed in large samples. In the limit, the '*t*-ratios' are merely rescaled to have unit variance.

However, how large is 'large'? The Monte Carlo distributions are plotted as histograms with the smoothed 'kernel density' curves superimposed. The normal p.d.f. with matching mean and variance is also shown for comparison. Estimator moments and upper tail quantiles are also reported in the experimental output. To compare the latter with the theoretical Student *t* or normal distributions, do as follows.

1. Choose Setup/Look Up Critical Value from the menu bar.
2. In the dialog, choose the Student t radio button.
3. Enter the degrees of freedom in the Denominator DF field. (e.g. enter 48 for the case $T = 50$).
4. Enter the tail probablity (say 0.05) in the text field, and press the Critical Value button.

When $\rho = 1$, the distributions are not normal even asymptotically, but correspond to one of the 'Dickey-Fuller' distributions. Tabulations of these, for selected probabilities, can be found in Appendix H of the TSM documentation.