in this case comes from the idea of analysing samples of progressively increasing size. The mathematical theory often does not distinguish between the types of sequence under consideration, and some of our definitions and results apply generally, but a clue to the usual application will be given by the choice of index symbol, t or n as the case may be.

A leading case which does not fall under the definition of a sequence is where $\mathbb{T}$ is partially ordered. When there are two dimensions to the observations, as in a panel data set having both a time dimension and a dimension over agents, x may be called a *random field*. Such cases are not treated explicitly here, although in many applications one dimension is regarded as fixed and the sequence notion is adequate for asymptotic analysis. However, cases where $\mathbb{T}$ is either the product set $\mathbb{Z} \times \mathbb{N}$, or a subset thereof, are often met below in a different context. A *triangular stochastic array* is a doubly-indexed collection of random variables,

$$\begin{pmatrix} X_{11} & X_{21} & X_{31} & \dots \\ X_{12} & X_{22} & X_{32} & \dots \\ \vdots & \vdots & \vdots & \\ X_{1k_1} & \vdots & \vdots & \\ & X_{1k_2} & \vdots & \\ & & X_{3k_3} & \\ & & & \ddots \end{pmatrix}, \tag{12.2}$$

compactly written as $\{\{X_{nm}\}_{m=1}^{k_n}\}_{n=1}^{\infty}$, where $\{k_n\}_{n=1}^{\infty}$ is some increasing integer sequence. Array notation is called for when the points of a sample are subjected to scale transformations or the like, depending on the complete sample. A standard example is $\{\{X_{nt}\}_{t=1}^{n}\}_{n=1}^{\infty}$, where $X_{nt} = X_t/s_n$, and $s_n^2 = \sum_{t=1}^{n}\mathrm{Var}(X_t)$, or some similar function of the sample moments from 1 to n.

12.2 Convergence of Stochastic Sequences

Consider the functional expression $\{X_n(\omega)\}_1^{\infty}$ for a random sequence on the space $(\Omega,\mathcal{F},P)$. When evaluated at a point $\omega \in \Omega$ this denotes a *realization* of the sequence, the actual collection of real numbers generated when the outcome ω is drawn. It is natural to consider in the spirit of ordinary analysis whether this sequence converges to a limit, say $X(\omega)$. If this is the case for every $\omega \in \Omega$, we would say that $X_n \to X$ *surely* (or elementwise) where, if X_n is an $\mathcal{F}/\mathcal{B}$-measurable r.v. for each n, then so is X, by **3.26**.

But, except by direct construction, it is usually difficult to establish in terms of a given collection of distributional properties that a stochastic sequence converges surely to a limit. A much more useful notion (because more easily shown) is *almost sure convergence*. Let $C \subseteq \Omega$ be the set of outcomes such