conditions elsewhere in the problem, so it is still desirable to develop the parallel results for the uniform case.

The strong restrictions needed to ensure processes are mixing, which these examples point to (to be explored further in the next section), threaten to limit the usefulness of the mixing concept. However, technical infringements like the ones demonstrated are often innocuous in practice. Only certain aspects of mixing, encapsulated in the concept of a *mixingale*, are required for many important limit results to hold. These are shared with so-called *near-epoch dependent functions* of mixing sequences, which include cases like **14.7**. The theory of these dependence concepts is treated in Chapters 16 and 17. While Chapter 15 contains some necessary background material for those chapters, the interested reader might choose to skip ahead at this point to find out how, in essence, the difficulty will be resolved.

14.4 Sufficient Conditions for Strong and Uniform Mixing

The problems in the counter-examples above are with the form of the marginal shock distributions – discrete or unbounded, as the case may be. For strong mixing, a degree of smoothness of the distributions appears necessary in addition to summability conditions on the coefficients of linear processes. Several sufficient conditions have been derived, both for general MA(∞) processes and for autoregressive and ARMA processes. The sufficiency result for strong mixing proved below is based on the theorems of Chanda (1974) and Gorodetskii (1977). These conditions are not the weakest possible in all circumstances, but they have the virtues of generality and comparative ease of verification.

14.9 Theorem Let $X_t = \sum_{j=0}^{\infty}\theta_j Z_{t-j}$ define a random sequence $\{X_t\}_{-\infty}^{\infty}$, where, for either $0 < r \le 2$ or r an even positive integer,

(a) Z_t is uniformly L_r-bounded, independent, continuous with p.d.f. f_{Z_t}, and

$$\sup_t \int_{-\infty}^{+\infty} \left| f_{Z_t}(z+a) - f_{Z_t}(z) \right| dz \le M|a|,\ M < \infty, \tag{14.42}$$

whenever $|a| \le \delta$, for some $\delta > 0$;

(b) $\sum_{t=0}^{\infty} G_t(r)^{1/(1+r)} < \infty$, where

$$G_t(r) = \begin{cases} 2\displaystyle\sum_{j=t}^{\infty}|\theta_j|^r, & r \le 2, \\ 2^{r-1}\left(\displaystyle\sum_{j=t}^{\infty}\theta_j^2\right)^{r/2}, & r \ge 2; \end{cases} \tag{14.43}$$

(c) $\theta(x) = \sum_{j=0}^{\infty}\theta_j x^j \ne 0$ for all complex numbers x with $|x| \le 1$.

Then $\{X_t\}$ is strong mixing with $\alpha_m = O\left(\sum_{t=m+1}^{\infty} G_t(r)^{1/(1+r)}\right)$. □

Before proceeding to the proof, we must discuss the implications of these three conditions in a bit more detail. Condition **14.9**(a) may be relaxed somewhat, as we