$$\le 2 \sup_{\boldsymbol{v}\in B'}\left\{\int_{\mathbb{R}^{m+k}}\left| f_W(\boldsymbol{w}+\boldsymbol{v}) - f_W(\boldsymbol{w})\right| d\boldsymbol{w}\right\}$$

$$= 2 \sup_{\boldsymbol{v}\in B'}\left\{\int_{\mathbb{R}^{m+k}}\left| \prod_{t=1}^{m+k} f_{Z_t}(z_t+\hat{v}_t) - \prod_{t=1}^{m+k} f_{Z_t}(z_t)\right| d\boldsymbol{z}\right\}$$

$$\le 2M \sup_{\boldsymbol{v}\in B'}\left\{\sum_{t=m+1}^{m+k}|\hat{v}_t|\right\}, \tag{14.58}$$

where it is understood in the final inequality (which is by **14.10**) that $|\hat{v}_t| \le \delta$ where δ is defined in condition **14.9**(a). The third equality substitutes $\hat{\boldsymbol{v}} = \boldsymbol{A}_{m+k}^{-1}\boldsymbol{v}$ and uses the fact that $\hat{\boldsymbol{v}}_1 = \boldsymbol{0}$ if $\boldsymbol{v}_1 = \boldsymbol{0}$ by lower triangularity of $\boldsymbol{A}_{m+k}$. For $\boldsymbol{v} \in B'$, note that

$$\sum_{t=m+1}^{m+k}|\hat{v}_t| = \sum_{t=m+1}^{m+k}\left|\sum_{j=0}^{t-m-1}\tau_j v_{t-j}\right|$$

$$\le \sum_{t=m+1}^{m+k}\left(\sum_{j=0}^{t-m-1}|\tau_j|\eta_{t-j}\right) \le \left(\sum_{j=0}^{\infty}|\tau_j|\right)\sum_{t=m+1}^{m+k}\eta_t, \tag{14.59}$$

assuming $\boldsymbol{\eta}$ has been chosen with elements small enough that the terms in parentheses in the penultimate member do not exceed δ. This is possible by condition **14.9**(c).

For the final step, choose r to be the largest order of absolute moment if this is does not exceed 2, and the largest even integer moment, otherwise. Then

$$P(E^c) = P(|\boldsymbol{V}_2| > \boldsymbol{\eta})$$

$$= P\left(\bigcup_{t=m+1}^{m+k}\{|V_t| > \eta_t\}\right)$$

$$\le \sum_{t=m+1}^{m+k}P(|V_t| > \eta_t) \le \sum_{t=m+1}^{m+k}E|V_t|^r\eta_t^{-r}, \tag{14.60}$$

by the Markov inequality, and

$$E|V_t|^r \le \sup_s E|Z_s|^r G_t(r), \tag{14.61}$$

where $G_t(r)$ is given by (14.43), applying **11.15** for $r \le 2$ (see (11.65) for the required extension) and Lemma **14.11** for $r > 2$. Substituting inequalities (14.58), (14.59), (14.60), and (14.61) into (14.56) yields

$$|P(G\cap H) - P(G)P(H)| \ll \sum_{t=m+1}^{m+k}\left(\eta_t + G_t(r)\eta_t^{-r}\right). \tag{14.62}$$